

Memory Based_JEE Main Online Test_07-01-20_Evening PHYSICS

1. An electron of mass m and a photon have nearly equal energy then find $\frac{\lambda_e}{\lambda_p} = ?$

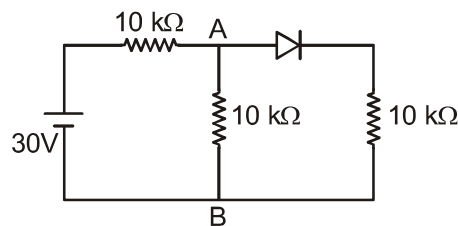
Sol. $\lambda_e = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$$\lambda_p = \frac{hc}{E}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{E}{c} \frac{1}{\sqrt{2mE}}$$

$$\frac{\lambda_e}{\lambda_p} = \frac{1}{c} \sqrt{\frac{E}{2m}}$$

2. Find out potential difference across AB.

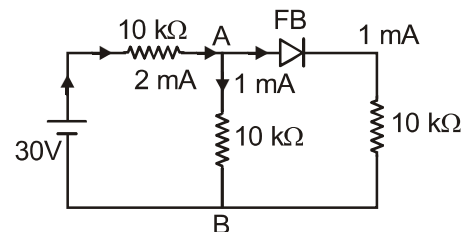


- Sol.** Diode is forward Bias

$$i = \frac{30}{15 \times 10^3} = 2 \text{ mA (from battery)}$$

This current divides equally in both resistor of resistance $10 \text{ k}\Omega$ which is 1 mA

$$(V_A - V_B) = \left(10 \times 10^3 \times \frac{1}{10^3} \right) = 10 \text{ volt}$$



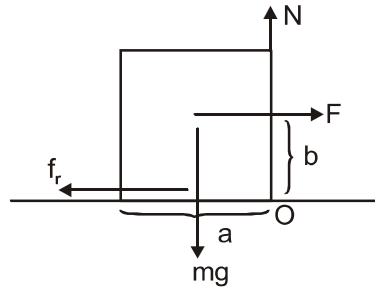
3. In YDSE experiment distance between the slits and screen $D = 1.5 \text{ m}$, distance between the slits $d = 0.15 \text{ mm}$ and wavelength $\lambda = 580 \text{ nm}$ of light find out fringe width.

Sol. $\beta = \frac{\lambda D}{d} \Rightarrow \beta = \frac{589 \times 10^{-9} \times 1.5}{0.15 \times 10^{-3}}$

$$\beta = 589 \times 10^{-5} \text{ m}$$

$$\beta = 5.89 \text{ mm}$$

4. A cubical block of side length a is kept on a rough surface with friction coefficient $\mu = 0.4$. A force F is acting at a height b above the surface as shown in the figure then, find ratio of a/b such that block is just about to topple?

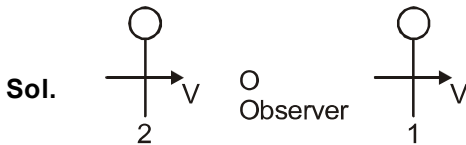


Sol. τ about O; $Fb = mg \cdot \frac{a}{2} \Rightarrow \frac{F}{mg} = \frac{a}{2b}$

$$F = \mu mg \Rightarrow \frac{F}{mg} = 0.4$$

$$\frac{a}{2b} = \frac{4}{10} \Rightarrow \frac{a}{b} = \frac{4}{5}$$

5. A stationary observer 'S'. One tuning fork is coming towards and moving away from the stationary observer with same velocity 'v'. Observer is hearing 2 beats.
(Frequency of the tuning fork = 1400 Hz) (Velocity of sound = 350 m/s)



$$F_1 = \left(\frac{V_s}{V_s + V} \right) F_0$$

$$F_2 = \left(\frac{V_s}{V_s - V} \right) \times F_0$$

$$f_2 - f_1 = 2$$

$$V_s f_0 V = V_s^2 - V^2$$

$$V^2 + 490,000 V - 122500 = 0$$

$$V = 0.25 \text{ m/s}$$

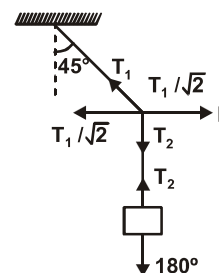
$$V = \frac{1}{4} \text{ m/s}$$

6. A 10 kg mass is hanging from a 4m long rope attached to a rigid support when a force F is applied at the mid-point of the rope the upper half of the rope forms an angle of 45° from the vertical then find F .

Sol. Here, according to figure,

$$T_2 = 100 = \frac{T_1}{\sqrt{2}}$$

$$\text{and } F = \frac{T_1}{\sqrt{2}} = 100 \text{ N}$$



7. Find dimensional formula of energy density = $\frac{1}{2} B^2/\mu_0$

Sol. Energy density = $\frac{1}{2} \frac{B^2}{\mu_0}$

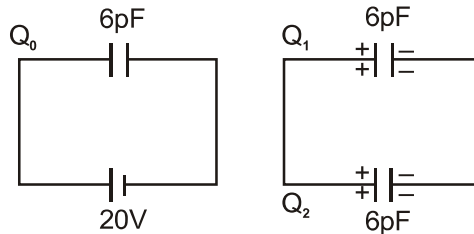
$$\left[\frac{B^2}{2\mu_0} \right] = \left[\frac{E}{V} \right] = \frac{ML^2T^{-2}}{L^3} = ML^{-1}T^{-2}$$

8. A lens ($\mu = 1.5$) is placed in water ($\mu = 1.42$) then find the ratio of focal length in water and air.

Sol. $\frac{1}{f_a} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$
 $\frac{1}{f_w} = \left(\frac{\mu_L}{\mu_w} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$

$$\frac{f_w}{f_a} = \left(\frac{\frac{3}{2} - 1}{\frac{1.5}{1.42} - 1} \right) = \frac{0.5}{0.056} = 8.92$$

9. A 6pF capacitor is charged by a 20V supply. It is then disconnected from the supply and is connected to another uncharged 6pF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat.



Sol. $C_1 = 6\text{pF}$ $V_1 = 20\text{V}$
 $C_2 = 6\text{pF}$ $V_2 = 0\text{V}$

$$\Delta U = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

$$\Delta U = \frac{1}{2} \times \frac{6 \times 10^{-12} \times 6 \times 10^{-12}}{6 \times 10^{-12} + 6 \times 10^{-12}} (20 - 0)^2$$

$$\Delta U = \frac{1}{2} \times \frac{6 \times 10^{-12}}{12 \times 10^{-12}} \times 400$$

$$\Delta U = 6 \times 10^{-10} \text{ J}$$

10. Activity of a radioactive material is 700/sec after 30 min it becomes 500/sec then find half life of the material.

Sol. $A = A_0 e^{-\lambda t}$

$$\frac{500}{700} = e^{-\lambda t}$$

$$\lambda = \frac{\log(7/5)}{30}$$

$$T_{1/2} = \frac{30 \log(2)}{\log(7/5)} \approx 62 \text{ min}$$

11. A non-uniform disc of radius R has surface mass density $\sigma = A + Br$ where r is distance from centre of disc. Then find moment of inertia of disc.

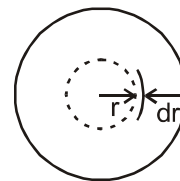
Sol. $dl = dm \cdot r^2$

$$= \sigma 2\pi r dr \cdot r^2$$

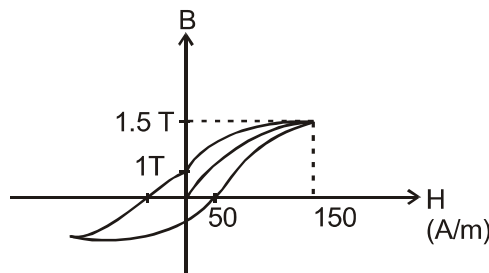
$$= \int_0^R (A + Br) 2\pi r^3 dr$$

$$I = \frac{A2\pi R^4}{4} + \frac{B2\pi R^5}{5}$$

$$I = 2\pi R^4 \left[\frac{A}{4} + \frac{BR}{5} \right]$$



12. For a given hysteresis curve for ferromagnetic material.



Find Coercivity, Retentivity and saturation magnetic field.

Sol. From the given graph:

Coercivity = 50 A/m

Retentivity = 1 T

Saturation M.F. = 1.5 T

13. m gram steam at 100°C is mixed with 200 gm ice at 0°C which results in water at 40°C Then find the m.

Sol. Law of Calorimetry \Rightarrow Heat Lost = Heat gained.

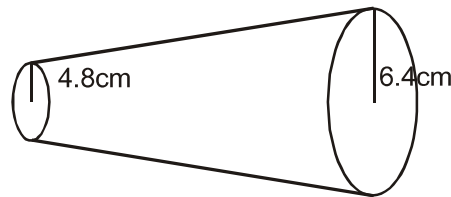
$$m \times 540 + m \times 1 \times 60 = 200 \times 80 + 200 \times 1 \times 40$$

$$\Rightarrow 60 m = 1600 + 800$$

$$\Rightarrow m = \frac{2400}{60}$$

$$\Rightarrow m = 40 \text{ gm}$$

14. Fluid is flowing through a conical tube shown below, the find the ratio of maximum velocity two the minimum velocity for the flow of the fluid.



Sol. $A_1 V_1 = A_2 V_2$

$$\pi r_1^2 V_1 = \pi r_2^2 V_2$$

$$\frac{V_1}{V_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{6.4}{4.8}\right)^2$$

15. The gravity at the North Pole is 10 m/s^2 . The weight of the object at North Pole is 196 N. Radius of the earth is 6400 km. Find the weight at the equator.

(A) 194.66

(B) 195.32

(C) 194.32

(D) 195

Ans. (B)

Sol. $g_{\text{North Pole}} = 10$

The weight of the object at North Pole = 196 N.

Radius of the earth = 6400 km.

at Equator:

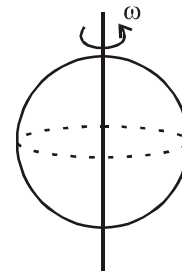
$$g' = g - \omega^2 R$$

$$W' = mg - m\omega^2 R$$

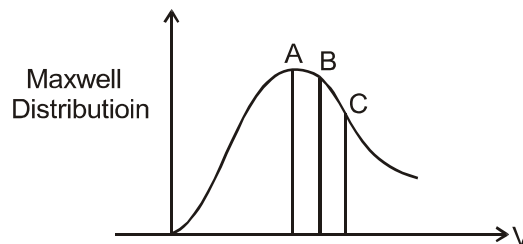
$$W' = 196 - 19.6 \left(\frac{2\pi}{T}\right)^2 \times 6400 \times 10^3$$

$$W' = 196 - 19.6 \frac{4 \times \pi^2}{(24 \times 3600)^2} \times 6400 \times 10^3$$

$$W' = 195.32$$



16. In the following Maxwell Distribution curve which point represent RMS, Average and most probable speed of the paticle.

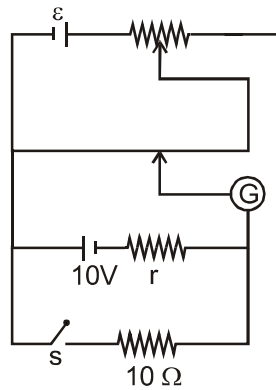


Sol. A → Most Probable

B → Average

C → RMS

17. In the given potentiometer arrangement null point occurs at 560 cm when switch S is open & at 500 cm when S is closed then find value of internal resistance r.



Sol. Let ϕ is potential gradient in potentiometer wire.

here, when switch s is open

$$\varepsilon = 560 \phi \quad \dots(i)$$

when s is closed

$$\varepsilon - ir = 500 \phi \quad \dots(ii)$$

where $i = \frac{\varepsilon}{10+r}$

Then, $\varepsilon - \frac{\varepsilon r}{10+r} = 500\phi \quad \dots(iii)$

Dividing equation (i) & (iii)

$$\Rightarrow \frac{\varepsilon}{\frac{\varepsilon(10+r-r)}{10+r}} = \frac{560\phi}{500\phi}$$

$$\Rightarrow \frac{10+r}{10} = \frac{28}{25}$$

$$\therefore r = \frac{6}{5}$$

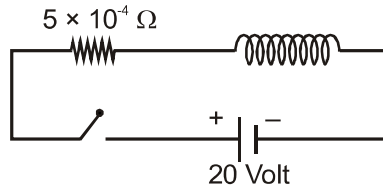
18. $E = E_0 \hat{i}$, $B = B_0 \hat{j}$, $V = V_0 \hat{j}$ Find the time, when speed becomes $2V_0$.



$$\sqrt{3}V_0 = \frac{qE}{m}t$$

$$\frac{\sqrt{3}mV_0}{qE} = t$$

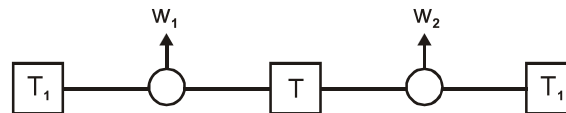
19. Find the ratio of current at $t = \infty \text{ sec}$ and $t = 40 \text{ sec}$. ($e^2 = 7.73$)



Sol.
$$\frac{i_{\infty}}{i_{(40\text{sec})}} = \frac{V/R}{\frac{V}{R} \left(1 - e^{-\frac{RT}{L}}\right)}$$

$$\frac{i_{\infty}}{i_{(40\text{sec})}} = \frac{1}{\left(1 - e^{-\frac{RT}{L}}\right)} = \frac{7.73}{(7.73 - 1)} \approx 1.15$$

20. Two reservoir with hot one at T_1 and cold one of 2nd reservoir at T_2 such that sink of 1st is source of second if work done by both the engine are equal ($w_1 = w_2$) then find temperature T.

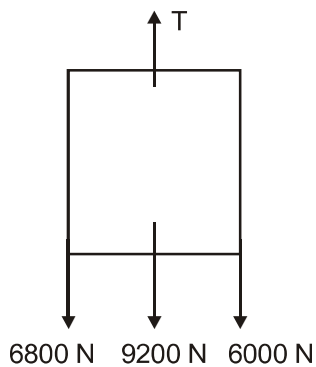


Sol. $w_1 = w_2$
 $T_1 - T = T - T_2$
 $T_1 + T_2 = 2T$

$$T = \left(\frac{T_1 + T_2}{2}\right)$$

21. A lift of mass 920 kg carries 10 persons each of mass 68 kg with velocity 3m/s and friction of 6000 N act opposite to motion then find power delivered by lift ?

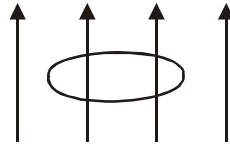
Sol.



$$\therefore T = 22000 \text{ N}$$

$$P = T \times V = 22000 \times 3 = 66000 \text{ Watts.}$$

22. A disc rotating in a uniform magnetic field of time period = 10 sec. then find maximum and minimum E.M.F



Sol. $\phi = BA \cos(\omega t)$

$$|\varepsilon| = \left| \frac{d\phi}{dt} \right| = BA \sin(\omega t)$$

Maximum E.M.F

$$\omega t = \frac{\pi}{2}$$

$$\frac{2\pi}{T} t = \frac{\pi}{2}$$

$$t = \frac{T}{4} = 2.5 \text{ sec}$$

Minimum E.M.F

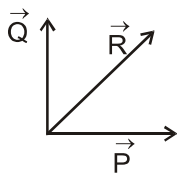
$$\omega t = \pi$$

$$\frac{2\pi}{T} t = \pi$$

$$t = \frac{T}{2} = 5 \text{ sec}$$

23. \vec{P} & \vec{Q} are 2 vectors and a third vector $\vec{R} = \vec{P} + \vec{Q}$ such that $|\vec{R}| = |\vec{P}|$. then angle between $2\vec{P} + \vec{Q}$ and \vec{Q} is = ?

Sol.



$$R^2 = P^2 + Q^2 + 2PQ \cos\theta$$

$$0 = Q(Q + 2P\cos\theta) = 0$$

$$Q = 0 \text{ (Not possible)}$$

$$Q + 2P\cos\theta = 0 \Rightarrow |Q + 2P\cos\theta = 0|$$

$$(2\vec{P} + \vec{Q}) \cdot \vec{Q}$$

$$2\vec{P} \cdot \vec{Q} + Q^2$$

$$2PQ \cos\theta + Q^2$$

$$= Q(2P\cos\theta + Q) = 0$$

Angle between $(2\vec{P} + \vec{Q})$ & \vec{Q} is 90°

Memory Based_JEE Main Online Test_07-01-20_Evening

CHEMISTRY

1. Among the given pair, the element with greater value of electron gain enthalpy (S, Se), (Cl, F), (Na, Li)

Ans. **S, Cl, Li**

2. For cyanide CN^- , the correct option is

Ans. **Bond order = 3, Diamagnetic**

3. In $[\text{Ma}_2\text{b}_2]$ complex, number of optical isomers for tetrahedral and square planar geometry is :

Ans. **optical isomers $sp^3 = 0$**

$dsp^2 = 0$

4. If cobalt (III), is approached by strong field ligand.

Ans. **Correct state of statement is to be selected**

5. $\text{Cl}_2 + \text{NaOH}$ (hot & conc.) \longrightarrow X + side product

$\text{Cl}_2 + \text{NaOH}$ (dry) \longrightarrow Y

Product X & Y respectively

Ans. **NaClO_3 , Ca(OCl)_2**

6. If melting point of metal is very low in comparison to impurities associated with it, then which methods is used for refining :

Ans. **Liquation**

7. Which of the following statements are **correct**

(I) Decomposition of hydrogen peroxide produces O_2 .

(II) 2-ethyl anthaquinol is used in preparation of H_2O_2 .

(III) H_2O_2 is used in preparation of sodium metaperborate.

(IV) Oxygen is released on thermal decomposition of KClO_3 , $\text{Pb(NO}_3)_2$, NaNO_3 .

Ans. **All the above statements are correct.**

8. Which reaction is redox in nature

(1) Formation of ozone.

(2) Reaction of dinitrogen and dioxygen to form nitric oxide

(3) Acid-Base reaction was given.

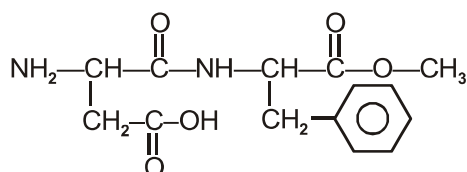
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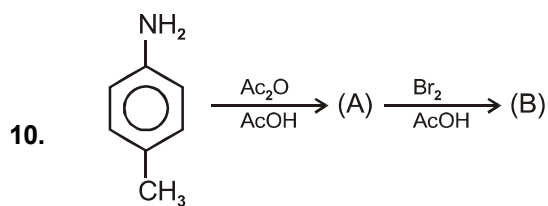
Ans. **Reaction of dinitrogen and dioxygen to form nitric oxide is redox reaction.**

9. Number of sp^2 carbons in Aspartame

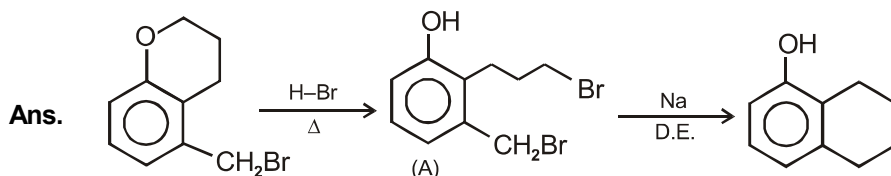
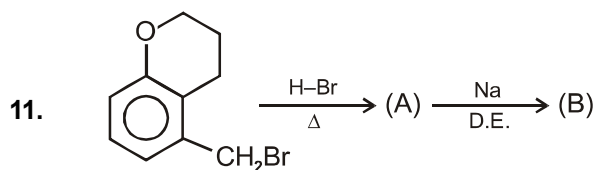
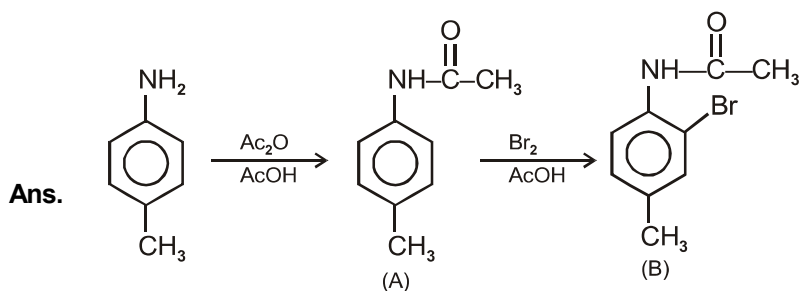
Ans. **9**

Sol.

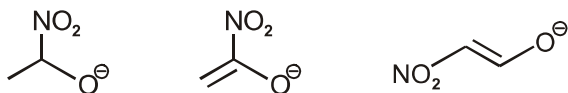




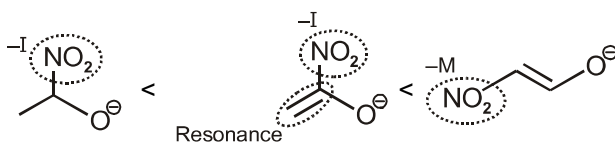
Find (B)

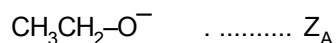
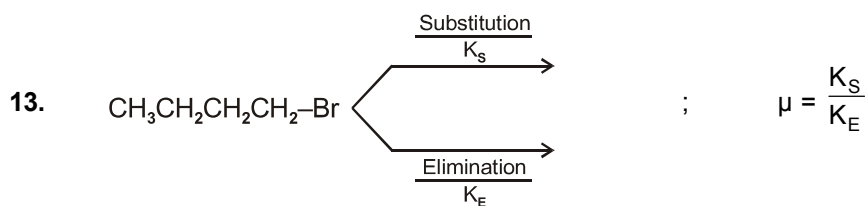


12. Find correct order of stability of following anions :



Sol. Stability of carbanion $\propto -M, -I, -H$

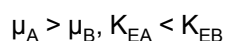




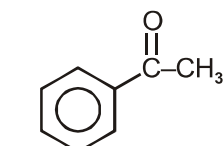
Identify correct option if given compound is react with Z_A and Z_B :

- (1) $\mu_A > \mu_B, K_{EA} > K_{EB}$ (2) $\mu_A < \mu_B, K_{EB} > K_{EA}$ (3) $\mu_A > \mu_B, K_{EA} < K_{EB}$ (4) $\mu_A < \mu_B, K_{EA} > K_{EB}$

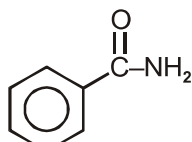
Ans. (3)



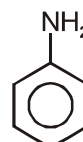
14. A column is packed with aniline, Acetophenon and Benzamide. Silica gel is taken as stationary phase with eluent hexane and ethyl acetate in 20 : 80 ratio. Identify the order from bottom to top.



Acetophenone



Benzamide



Aniline

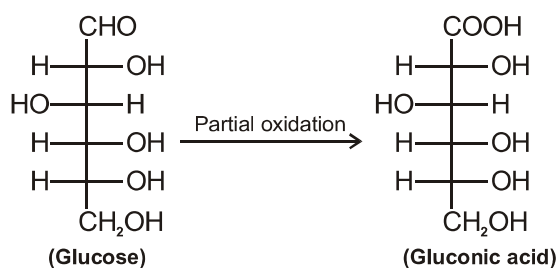
Ans. Acetophenone > Benzamide > Aniline (Bottom to Top)

15. Correct statement:

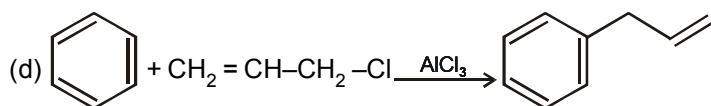
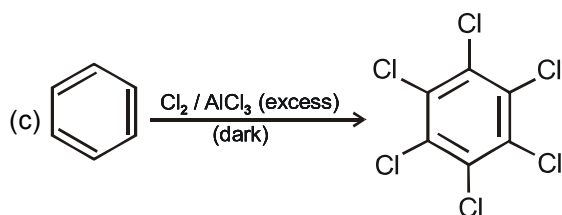
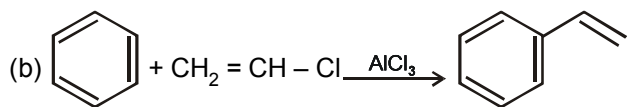
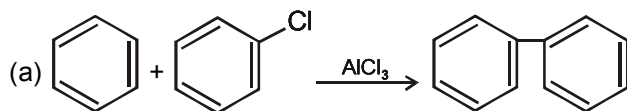
- (1) Glucose reaction with conc. HNO_3 to give gluconic Acid
(2) Gluconic Acid is a dicarboxylic acid
(3) Gluconic Acid is formed by partial oxidation of glucose
(4) Gluconic acid pyranose form is hemiacetal or ketal

Ans. (3)

Gluconic Acid is formed by partial oxidation of glucose.



16. Identify the correct reaction:



(1) a, b, c

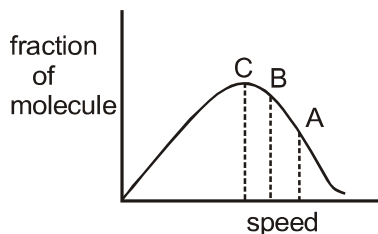
(2) b, c, d

(*3) c, d

(4) a, d, c

Ans. (3)

17. Find U_{rms} , U_{mps} , U_{avg} from the following curve.

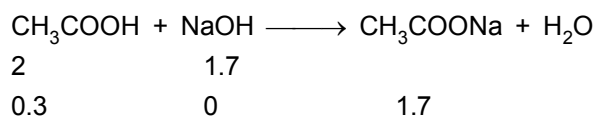


Identify U_{rms} , U_{mps} , U_{avg} = ?

Sol. $U_{\text{mps}} = C$ $U_{\text{avg}} = B$ $U_{\text{rms}} = A$

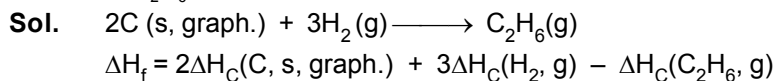
18. 3 gm of CH_3COOH is mixed with 0.1 M, 200 ml HCl solution. This solution is diluted up to 500 ml, 20 ml of the solution is reacted with 5 M, 0.5 ml NaOH solution. Find the pH of the final solution.

Sol. mmole of $\text{CH}_3\text{COOH} = 50$ in 500 mL solution
mmole of HCl = 20 in 500 mL solution
In 20 mL solution mmole of $\text{CH}_3\text{COOH} = 2$
mmole of NaOH = 2.5

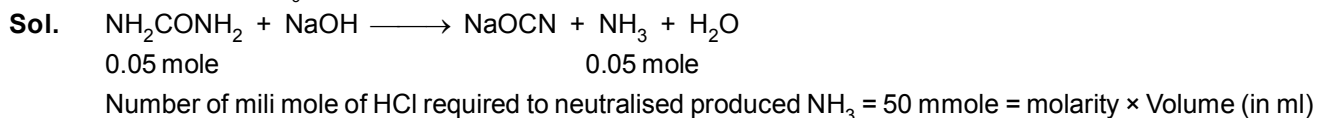


$$\text{pH} = 4.74 + \log \frac{1.7}{0.3} = 5.49$$

19. Enthalpy of combustion of C(s, graphite), H₂(g) and ethane (C₂H₆)(g) are given. Find the enthalpy of formation of C₂H₆(g).



20. 3 g of urea reacts with NaOH then NH₃ is released. How much volume of HCl solution is required neutralized the released NH₃.



21. Question from chemical kinetics based on reversible reaction to write rate law of backward reaction :

22. Question from surface chemistry to calculate Flocculation value for As₂S₃ sol.

23. Find out incorrect statement :

(1) $\Lambda_m^0(\text{H}_2\text{O}) = \Lambda_m^0(\text{NaOH}) + \Lambda_m^0(\text{HCl}) - \Lambda_m^0(\text{NaCl})$

(2) $\Lambda_m^0(\text{NaBr}) + \Lambda_m^0(\text{NaCl}) = \Lambda_m^0(\text{NaCl}) + \Lambda_m^0(\text{KBr})$

(3) $\Lambda_m^0(\text{NaCl}) - \Lambda_m^0(\text{NaBr}) = \Lambda_m^0(\text{KCl}) - \Lambda_m^0(\text{KBr})$

(4) $\Lambda_m^0(\text{NaCl}) + \Lambda_m^0(\text{KBr}) = \Lambda_m^0(\text{NaBr}) + \Lambda_m^0(\text{KCl})$

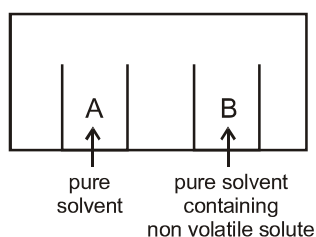
Ans. (2)

24. Which is redox reaction among the following :

- (1) reaction between N₂ and O₂ at 2000° C (2) reaction between [Co(NH₃)₆]Cl₃ and AgNO₃
 (3) reaction between H₂SO₄ and NaOH (4) formation of ozone in presence of sun light

Ans. (1)

25.



After some time, choose correct option :

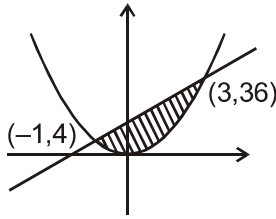
- (1) Volume of A and B remains same (2) Volume of A decreases and B same
 (3) Volume of A same and B decreases (4) Volume of A and B both decreases

Ans. (4)

Memory Based_JEE Main Online Test_07-01-20_Evening

MATHEMATICS

1. Consider the region bounded by $4x^2 \leq y \leq 8x + 12$
Find the area bounded = ?



$$\begin{aligned} \text{Hence, area} &= \int_{-1}^3 (8x + 12) - (4x^2) dx \\ &= \frac{148}{3} \end{aligned}$$

2. $3 + 4 + 8 + 9 + 13 + 14 + \dots$ (40 terms) = 102m
then m = ?

Sol. $7 + 17 + 27 + \dots$ (20 term)

$$\begin{aligned} &= \frac{20}{2} [(2 \times 7) + 19 \times 10] \\ &= 10[14 + 190] \Rightarrow m = 20 \end{aligned}$$

3. Tangents are drawn from $(0, 0)$ to $x^2 + y^2 - 8x + 4y + 16 = 0$ which touches the circle at A and B Find $(AB)^2$
Ans. 12.8

Sol. $BC = \sqrt{16 + 4 - 16} = 2$

equations of AB

$$x \cdot 0 + y \cdot 0 - 4(x + 0) + 2(y + 0) + 16 = 0$$

$$-4x + 2y + 16 = 0$$

$$-2x + y + 8 = 0$$

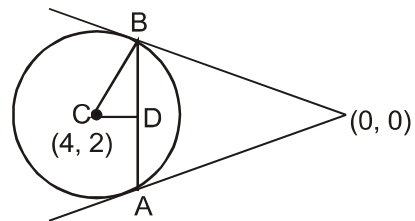
$$CD \text{ is } = \left| \frac{2 \times 4 - 2 - 8}{\sqrt{4 + 1}} \right| = \left| \frac{-8 - 2 + 8}{\sqrt{4 + 1}} \right| = \frac{2}{\sqrt{5}}$$

$$BD = \sqrt{(CB)^2 - (CD)^2} = \sqrt{4 - \frac{4}{5}} = \sqrt{\frac{16}{5}}$$

$$= \sqrt{8 - \frac{4}{5}} = \sqrt{\frac{24}{5}} \sqrt{\frac{36}{5}} = \frac{4}{\sqrt{5}}$$

$$AB = 2 \times BD = 8\sqrt{5}$$

$$AB^2 = \frac{64}{5} = 12.8$$



4. There are five machines, prob. of failing of any one of machine on a particular day is $\frac{1}{4}$. The probability that at most two machines are broken is $\left(\frac{3}{4}\right)^3 k$ find k

Sol. $q = \frac{1}{4}$

$$\Rightarrow P = \frac{3}{4}$$

$$\text{Given } {}^5C_0 p^5 + {}^5C_1 qp^4 + {}^5C_2 q^2 p^3 = \frac{27}{64} k$$

$$k = \frac{34}{16} = \frac{17}{8}$$

5. If tangent on $\frac{x^2}{a^2} + \frac{y^2}{9} = 1$ is $3x + 4y = 12\sqrt{2}$ find distance between foci.

Ans. $2\sqrt{7}$

Sol. \therefore Tangent is $y = -\frac{3x}{4} = 3\sqrt{2}$

$$m = -\frac{3}{4}, C = 3\sqrt{2}$$

$$c^2 = a^2 m^2 + b^2$$

$$18 = \frac{9a^2}{16} + 9$$

$$9 = \frac{9a^2}{16}$$

$$a = 4$$

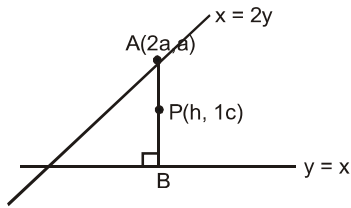
$$9 = 16(1 - e^2)$$

$$e = \frac{\sqrt{7}}{4}$$

$$\text{Distance between foci} = 2ae = 2\sqrt{7}$$

6. Find locus of mid point of perpendicular drawn from $x = 2y$ to $y = x$

Sol.



Equation of AB : $x + y = 3a$

B is P.O.I $\begin{cases} y = x \\ x + y = 3a \end{cases} \Rightarrow B \left(\frac{3a}{2}, \frac{3a}{2} \right)$

$h = 2a + \frac{3a}{2}$

$k = a + \frac{3a}{2}$

$\frac{h}{k} = \frac{7}{5}$

$y = \frac{5}{7}x$

7. If $f(x) = x^3 - 4x^2 + 8x + 12$
find 'C' by using L.M.V.T if $x \in [0, 1]$

Sol. $f'(C) = 3C^2 - 8C + 8 = \frac{f(1) - f(0)}{1 - 0}$

$3C^2 - 8C + 8 = 5$

$3C^2 - 8C + 3 = 0$

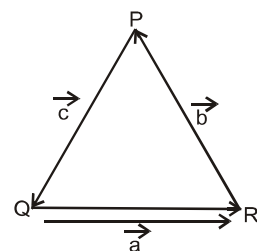
$C = \frac{4 - \sqrt{7}}{3}$

8. Let $\vec{a}, \vec{b}, \vec{c}$ are unit vectors satisfying $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $\vec{d} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ and $\lambda = \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ then
 \vec{d} and λ .

Sol. ΔPQR is equilateral Δ .

$\vec{d} = 3\vec{a} \times \vec{b}$ and

$\lambda = 3(\vec{a} \cdot \vec{b}) = 3\cos 120^\circ = -\frac{3}{2}$



9. Find coefficient of x^7 in S where S is
 $S = (1 + x)^{10} + x(1 + x)^9 + x^2(1 + x)^8 + \dots + x^{10}$
(1) 120 (*) 330 (3) 240 (4) 420

Ans. (2)

Sol. Coefficient is $= {}^{10}C_7 + {}^9C_6 + \dots + {}^3C_3$
 $= {}^3C_3 + {}^4C_3 + {}^5C_3 + \dots + {}^{10}C_3$
 $= {}^4C_4 + {}^4C_3 + {}^5C_3 + \dots + {}^{10}C_3$
 $= {}^{11}C_4$ using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

10. Let $k \in I$; $r \in I$ and $6 \cdot {}^{35}C_r = (k^2 - 3) \cdot {}^{36}C_{r+1}$. Find number of all ordered pair (r, k)

Ans. 4

Sol. $6 \cdot {}^{35}C_r = (k^2 - 3) \frac{36}{r+1} \cdot {}^{35}C_r$

$$k^2 - 3 = \frac{r+1}{6}$$

$$r = 6k^2 - 19$$

$$\text{as } r \in [0, 35]$$

$$k = \pm 2, \pm 3$$

11. If the roots of $x^2 - x - 1 = 0$ are α and β ; and $P(k) = \alpha^k + \beta^k$; then which of the following is incorrect .

(1) $P(5) = P(2) \cdot P(3)$

(2) $P(1) + P(2) + P(3) + P(4) + P(5) = 26$

(3) $P(5) = 11$

(4) $P(5) = P(4) + P(3)$

Sol. $P(1) = 1$

$$P(2) = (\alpha + \beta)^2 - 2\alpha\beta = 3$$

$$P(3) = \alpha^3 + \beta^3$$

$$= \alpha(\alpha + 1) + \beta(\beta + 1)$$

$$= 3 + 1 = 4$$

$$P(4) = \alpha^4 + \beta^4 = (\alpha^2 + \beta^2) - 2\alpha^2\beta^2$$

$$= 9 - 52 = 7$$

$$P(5) = \alpha^5 + \beta^5 = \alpha(\alpha^4) + \beta(\beta^4)$$

$$= \alpha(\alpha + 1)^2 + \beta(\beta + 1)^2$$

$$= \alpha(\alpha^2 + 2\alpha + 1) + \beta(\beta^2 + 2\beta + 1)$$

$$= 4 + 2(3) + 1$$

$$= 11$$

12. Let $A = [a_{ij}]_{3 \times 3}$ be square matrix of 3×3 .

Let $B = [b_{ij}]_{3 \times 3}$ be a matrix such that

$$b_{ij} = 3^{i+j-2} a_{ji}$$

if $|B| = 81$ find $|A|$.

Sol. $|B| = \begin{vmatrix} a_{11} & 3a_{21} & 3^2a_{31} \\ 3a_{12} & 3^2a_{22} & 3^3a_{32} \\ 3^2a_{13} & 3^3a_{23} & 3^4a_{33} \end{vmatrix}$

$$|B| = 3^3 \cdot 3^3 |A|$$

$$|B| = 81 = 3^6 |A|$$

$$|A| = \frac{1}{9}$$

13. Let $f(x)$ be a polynomial of degree 5 if $\lim_{x \rightarrow 0} \left(2 + \frac{f(x)}{x^3} \right) = 4$. Let $x = \pm 1$

are critical points, then which of the following is incorrect.

- (1) $f(1)$ is maxima and $f(-1)$ is minima. (*2) $f(1)$ is minima and $f(-1)$ is maxima.
(3) $f(1) - 4f(-1) = 4$ (4) $f(x)$ is odd function.

Ans. (2)

Sol. $\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 2$

$\therefore f'(x) = kx^2(x^2 - 1)$

$f(x) = k \left(\frac{x^5}{3} - \frac{x^3}{3} \right) + C$

$f(0) = 0 \Rightarrow C = 0$

$\lim_{x \rightarrow 0} \frac{f(x)}{x^3} = 2$

$\Rightarrow k \left(-\frac{1}{3} \right) = 2$

$k = -6$

$f(x) = 2x^3 - \frac{6}{5}x^5$

14. If $y\sqrt{1-x^2} = k - x\sqrt{1-y^2}$

and $y \left(\frac{1}{2} \right) = -\frac{1}{4}$, find $\frac{dy}{dx}$ at $x = \frac{1}{2}$

Sol. Let $x = \sin\alpha$

$y = \sin\beta$

$\therefore \sin(\alpha + \beta) = k$

$\alpha + \beta = \sin^{-1}k$

or $\sin^{-1}x + \sin^{-1}y = \sin^{-1}k$

$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} y' = 0$

$\Rightarrow y' = -\sqrt{\frac{1-y^2}{1-x^2}}$

$\therefore (y')_{x=\frac{1}{2}} = -\sqrt{\frac{1-\frac{1}{16}}{1-\frac{1}{4}}} = -\sqrt{\frac{15}{12}} = -\sqrt{\frac{5}{4}} = -\frac{\sqrt{5}}{2}$

15. Let $a_1 a_2 \dots a_n$ be a G.P. If $a_1 < 0$ and $a_1 + a_2 = 4$ and $a_3 + a_4 = 16$. If $a_1 + a_2 + \dots + a_9 = 4\lambda$
Find λ ?

Sol. $a + ar = 4$
 $ar^2 + ar^3 = 16$
 Divide : $r^2 = 4$
 $r = 2$ or -2
 $a = -4$ and $r = -2$

$$\text{Sum} = a \left(\frac{r^9 - 1}{r - 1} \right) = -4 \left(\frac{(-2)^9 - 1}{-3} \right)$$

$$= -\frac{4}{3} (2^9 + 1) = 4\lambda$$

$$\lambda = -\frac{513}{3} = -171$$

16. If $4\alpha \int_{-1}^2 e^{-\alpha|x|} dx = 5$

find α

Sol. $4\alpha \left(\int_{-1}^0 e^{\alpha x} dx + \int_0^2 e^{-\alpha x} dx \right) = 5$

$$4 \left(e^{\alpha x} \Big|_{-1}^0 - e^{-\alpha x} \Big|_0^2 \right) = 5$$

$$4 \left[(1 - e^{-\alpha}) - (e^{-2\alpha} - 1) \right] = 5$$

$$4e^{-2\alpha} + 4e^{-\alpha} - 3 = 0$$

$$e^{-\alpha} = -\frac{3}{2} \text{ or } \frac{1}{2}$$

$$\alpha = \ln 2$$

17. Solution of differential equation $(y^2 - x) \frac{dy}{dx} = 1$ cuts the x axis at A. Find A. (where $y(0) = 1/2$)

Sol. $\frac{dx}{dy} + x = y^2$

$$x \cdot e^y = \int y^2 e^y dy$$

$$x e^y = e^y \cdot y^2 - 2(y e^y) + 2e^y + c$$

$$x = y^2 - 2y + 2 + c e^{-y}$$

$$0 = \frac{1}{4} - 1 + 2 + c e^{-\frac{1}{2}} \quad (\text{use } y(0) = \frac{1}{2})$$

$$C = -\frac{5}{4} \sqrt{e}$$

$$A : y = 0 \Rightarrow x = 2 + c$$

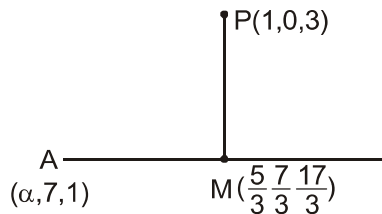
$$x = 2 - \frac{5}{4} \sqrt{e}$$

Subjective :

18. The foot of perpendicular from point P(1, 0, 3) on a line is $\left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$. If line passes through $(\alpha, 7, 1)$, find value of α .

Ans. 4

Sol.



$AM \perp PM$

$$\left(\alpha - \frac{5}{3}\right)\left(1 - \frac{5}{3}\right) + \left(7 - \frac{7}{3}\right)\left(0 - \frac{7}{3}\right) + \left(1 - \frac{17}{3}\right)\left(3 - \frac{17}{3}\right) = 0$$

$$\Rightarrow \alpha = 4$$

19.
$$\begin{cases} \frac{1}{x} \log \frac{(1+3x)}{1-2x} & x \neq 0 \\ K & x = 0 \end{cases}$$

is continuous at $x = 0$, Find K -

Ans. 5

Sol. $f(0) = K = \lim_{x \rightarrow 0} \frac{\log \left(\frac{1+3x}{1-2x} \right)}{x}$

$$= \lim_{x \rightarrow 0} \frac{\log(1+3x) - \log(1-2x)}{x}, \frac{0}{0}$$

by using L' Hospital

$$= \lim_{x \rightarrow 0} \frac{1 \times 3 - 1(-2)}{1+3x - 1-2x}$$

$$= \frac{3}{1+0} + \frac{2}{1-0} = 5$$

20. If mean and variance of the data 3, 7, 9, 12, 13, 20, x , y are 10 and 25 respectively ; find x, y .

Sol. $x + y = 16$ (As $\bar{x} = 10$)
& $x^2 + y^2 = 148$ (As $\sigma^2 = 25$)
 $\therefore xy = 54$

21. $X = \{x | x \in \mathbb{N} ; \cap \in [1, 50]$
 $A = \{n | n = 2\lambda ; \lambda \in \mathbb{N}\}$
 $B = \{m | m = 7\lambda ; \lambda \in \mathbb{N}\}$

Find minimum number of elements in subset of X such that it contains all elements of both A & B .

Sol. $A = \{2, 4, 6, \dots, 50\}$
 $B = \{7, 14, \dots, 49\}$
 $A \cap B = \{14, 28, 42\}$

Hence minimum number of elements = 3.

22. $x + y + z = 6$
 $x + 2y + 3z = 10$
 $3x + 2y + \lambda z = \mu$ has more than two solution .Find $\mu - \lambda^2$.

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 2 & \lambda \end{vmatrix} = 0$

$$\text{and } D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 3 & 2 & \mu \end{vmatrix} = 0$$

$$\mu = 14$$

$$\mu - \lambda^2 = 14 - 1 = 13$$