MATHEMATICS - CET 2020 - VERSION Code - D-2
SOLUTION

1. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{3-y}{-4}=\frac{z-4}{5}$ and the plane 2x – 2y + z = 5 is

(A) $\frac{3}{50}$ (B) $\frac{4}{5\sqrt{2}}$ (C) $\frac{\sqrt{2}}{10}$ (D) $\frac{3}{\sqrt{50}}$

Ans (C)

Given line is $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$

and plane is 2x – 2y + z = 5

w.k.t. $\sin\theta=\left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$

$\sin\theta=\frac{\left(3\hat{i}+4\hat{j}+5\hat{k}\right).\left(2\hat{i}-2\hat{j}+\hat{k}\right)}{\sqrt{9+16+25}\ \sqrt{4+4+1}}$

$=\frac{6-8+5}{\sqrt{50}\ \sqrt{9}}=\frac{3}{5\sqrt{2}.3}=\frac{1}{5\sqrt{2}}\ =\frac{1}{5\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}}{10}$

2. If a line makes an angle of $\frac{\pi}{3}$ with each of x and y-axis, then the acute angle made by z-axis is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) $\frac{\pi}{4}$

Ans (D)

Given $\alpha=\beta=\frac{\pi}{3}\quad\gamma=?$

$\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$

$\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2+\cos^2\gamma=1$

$\frac{1}{4}+\frac{1}{4}+\cos^2\gamma=1$

$\cos^2\gamma=\frac{1}{2}\Rightarrow\cos\gamma=\pm\frac{1}{\sqrt{2}}$

$\Rightarrow\gamma=\frac{\pi}{4}\quad[\because\gamma\text{ is acute}]$

3. The distance of the point (1, 2, –4) from the line $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}$ is

(A) $\frac{\sqrt{293}}{7}$ (B) $\frac{293}{49}$ (C) $\frac{\sqrt{293}}{49}$ (D) $\frac{293}{7}$

Ans (A)

Given point is (1, 2, –4) and the line is $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}$

$\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}=K$

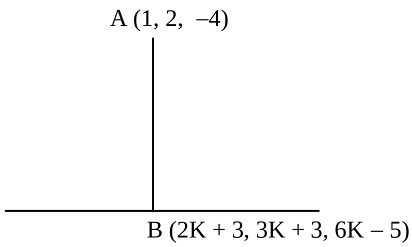
$\therefore$ Dr's of the line are (2K + 3, 3K + 3, 6K – 5)

Dr's of AB (2K + 2, 3K + 1, 6K – 1)

Since AB is perpendicular to the given line (2K + 2) 2 + (3K + 1)3 + (6K –1) 6 = 0

4K + 4 + 9K + 3 + 36 K – 6 = 0

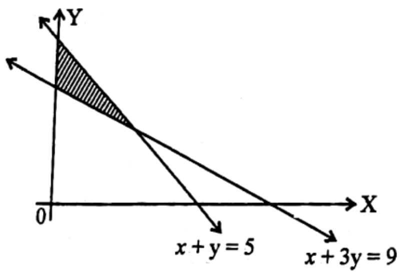
49K = – 1 $\quad K = \frac{-1}{49}$



$\therefore$ distance $= \sqrt{\left(\frac{96}{49}\right)^2 + \left(\frac{46}{49}\right)^2 + \left(\frac{43}{49}\right)^2}$

$= \sqrt{\frac{9216 + 2116 + 3025}{(49)^2}} \quad \sqrt{\frac{14357}{49 \times 49}} = \frac{\sqrt{293}}{7}$

4. The feasible region of an LPP is shown in the figure. If Z = 11x + 7y, then the maximum value of Z occurs at



(A) (3, 3) (B) (5, 0) (C) (3, 2) (D) (0, 5)

Ans (C)

The corner points are (0, 5), (0, 3), (3, 2)

At (0, 5) Z = 35

At (0, 3) Z = 21

At (3, 2) Z = 47

5. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let z = px + qy, where p, q > 0. Condition on p and q so that the minimum of z occurs at (3, 0) and (1, 1) is

(A) $p = \frac{q}{2}$ (B) p = 3q (C) p = q (D) p = 2q

Ans (A)

Given corner points are (0, 3), (1, 1), (3, 0)

z = px + qy

At (3, 0) z = 3p

At (1, 1) z = p + q

$\Rightarrow$ 3p = p + q

$2p = q \Rightarrow p = \frac{q}{2}$

6. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{6}$, then $P\left(\frac{A'}{B}\right)$ is

(A) $\frac{1}{3}$ (B) $\frac{1}{2}$ (C) $\frac{1}{12}$ (D) $\frac{2}{3}$

Ans (D)

Given $P(A)=\frac{1}{3}$ $\quad P(B)=\frac{1}{2}$ $\quad P(A\cap B)=\frac{1}{6}$

$P(A'|B)=1-P(A|B)$

$$=1-\frac{P(A\cap B)}{P(B)}=1-\frac{1}{3}=\frac{2}{3}$$

7. A die is thrown 10 times, the probability that an odd number will come up atleast one time is

(A) $\frac{1023}{1024}$ (B) $\frac{11}{1024}$ (C) $\frac{1013}{1024}$ (D) $\frac{1}{1024}$

Ans (A)

Given n = 10 $\quad p=\frac{1}{2},\quad q=\frac{1}{2}$

Required probability = 1 – P(X = 0)

$$=1-{}^{10}C_0\left(\frac{1}{2}\right)^{10-0}\left(\frac{1}{2}\right)^{0}$$

$$=1-\frac{1}{2^{10}}=1-\frac{1}{1024}$$

$$=\frac{1023}{1024}$$

8. The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2},\frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is

(A) $\frac{1}{4}$ (B) $\frac{1}{24}$ (C) $\frac{1}{8}$ (D) $\frac{1}{12}$

Ans (A)

Required probability = P(ABC′) + P(AB′C) + P(A′BC)

$$=\frac{1}{2}.\frac{1}{4}.\frac{2}{3}+\frac{1}{2}.\frac{3}{4}.\frac{1}{3}+\frac{1}{2}.\frac{1}{4}.\frac{1}{3}$$

$$=\frac{1}{12}+\frac{1}{8}+\frac{1}{24}=\frac{2+3+1}{24}=\frac{1}{4}$$

9. Events E_1 and E_2 form a partition of the sample space S. A is any even such that $P(E_1)=P(E_2)=\frac{1}{2}$, $P(E_2|A)=\frac{1}{2}$ and $P(A|E_2)=\frac{2}{3}$, then $P(E_1|A)$ is

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

Ans (D)

Given $P(E_1)=P(E_2)=\frac{1}{2}$

$P(E_2|A)=\frac{1}{2}$ $\quad P(A|E_2)=\frac{2}{3}$

Using Baye's theorem $P(E_2|A)=\dfrac{P(E_2)P(A|E_2)}{P(E_1)\,P(A|E_1)+P(E_2)\,P(A|E_2)}$

$$\frac{1}{2}=\frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)x+\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)} \Rightarrow \frac{1}{2}=\frac{\frac{1}{3}}{\frac{x}{2}+\frac{1}{3}}$$

$$\frac{x}{2}+\frac{1}{3}=\frac{2}{3} \Rightarrow \frac{x}{2}=\frac{1}{3} \Rightarrow x=\frac{2}{3}$$

$$P(E_1|A)=\frac{P(E_1)\,P(A|E_1)}{P(E_1)P(A|E_1)+P(E_2)\,P(A|E_2)}$$

$$=\frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)}=\frac{1}{2}$$

10. The value of $\sin^2 51° + \sin^2 39°$ is

(A) 0 (B) $\sin 12°$ (C) $\cos 12°$ (D) 1

Ans (D)

$\sin^2 51° + \sin^2 39° = \cos^2 39° + \sin^2 39° = 1$

11. If $\tan A + \cot A = 2$, then the value of $\tan^4 4 + \cot^4 A =$

(A) 1 (B) 4 (C) 5 (D) 2

Ans (D)

$\tan x + \cot x = 2$

$\tan^2 x + \cot^2 x + 2 \tan x \cot x = 2^2$

$\tan^2 x + \cot^2 x = 2$

$\tan^4 x + \cot^4 x + 2 \tan^2 x \cot^2 x = 4$

$\tan^4 x + \cot^4 x = 2$

12. If A = {1, 2, 3, 4, 5, 6}, then the number of subsets of A which contain atleast two elements is

(A) 63 (B) 57 (C) 58 (D) 64

Ans (B)

Subsets of A are $2^6 = 64$

Subsets of A which contain atleast two elements = 64 – 7 = 57

13. If n(A) = 2 and total number of possible relations from set A to set B is 1024, then n(B) is

(A) 20 (B) 10 (C) 5 (D) 512

Ans (C)

n(A) = 2

$2^{mn} = 1024$

$(2^2)^n = 2^{10}$

$2^{2\times5} = 2^{10}$

n(B) = 5

14. The value of

$^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

(A) 1 (B) $^{17}C_{10}$ (C) $^{17}C_3$ (D) 0

Ans (D)

$^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7 = {}^{17}C_{10} - {}^{17}C_7 = {}^{17}C_{10} - {}^{17}C_{10} = 0$

15. The number of terms in the expansion of $(x + y + z)^{10}$ is

(A) 142 (B) 11 (C) 110 (D) 66

Ans (D)

Number of terms in the expansion of $(x + y + z)^{10} = {}^{10+3-1}C_{10} = {}^{12}C_{10} = \frac{12!}{2!10!} = 66$

16. If $P(n) : 2^n < n!$

Then the smallest positive integer for which P(n) is true if

(A) 3 (B) 4 (C) 5 (D) 2

Ans (B)

$P(n) : 2^n < n!$

$n = 4, 2^4 < 4!$

$n = 4$

17. If z = x + iy, then the equation | z + 1 | = | z – 1| represents

(A) a parabola (B) x-axis (C) y-axis (D) a circle

Ans (C)

$|z + 1| = |z - 1|$

$|x + iy + 1| = |x + iy - 1|$

$\sqrt{(x+1)^2 + y^2} = \sqrt{(x-1)^2 + y^2}$

$(x + 1)^2 + y^2 = (x - 1)^2 + y^2$

$x^2 + 2x + 1 = x^2 + 1 - 2x$

$4x = 0$

$x = 0$

y-axis

18. If the parabola $x^2 = 4ay$ passes through the point (2, 1), then the length of the latus rectum is

(A) 4 (B) 2 (C) 8 (D) 1

Ans (A)

$x^2 = 4ay$ …(1)

equation (1) passing through (2, 1)

$4 = 4a(1)$

$a = 1$

length of latus rectum = 4a = 4(1) = 4

19. If the sum of n terms of an A.P. is given by $S_n = n^2 + n$, then the common difference of the A.P. is

(A) 1 (B) 2 (C) 6 (D) 4

Ans (B)

$S_n = n^2 + n$

$S_1 = 1 + 1 = 2 = T_1$

$S_2 = 2^2 + 2 = 6 = T_1 + T_2$

$T_2 = S_2 - S_1 = 6 - 2 = 4$

$d = T_2 - T_1 = 4 - 2 = 2$

20. The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if

(A) $lm' = ml'$ (B) $lm + l'm' = 0$ (C) $lm' + ml' = 0$ (D) $ll' + mm' = 0$

Ans (D)

The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if $ll' + mm' = 0$

21. The standard deviation of the data 6, 7, 8, 9, 10 is

(A) $\sqrt{10}$ (B) 2 (C) 10 (D) $\sqrt{2}$

Ans (D)

$$\bar{x} = \frac{6+7+8+9+10}{5} = \frac{40}{5} = 8$$

$$\sigma = \sqrt{\frac{1}{5}[4+1+0+1+4]} \qquad \because \quad \sigma = \sqrt{\frac{1}{n}\left(x_i - \bar{x}\right)^2}$$

$$\sigma = \sqrt{\frac{10}{5}} = \sqrt{2}$$

22. $\lim_{x\to 0}\left(\frac{\tan x}{\sqrt{2x+4-2}}\right)$ is equal to

(A) 3 (B) 4 (C) 6 (D) 2

Ans (D)

$\lim_{x\to 0}\left(\frac{\tan x}{\sqrt{2x+4}-2}\right)$ L H' Rule

$$= \lim_{x\to 0}\left(\frac{\sec^2 x}{\frac{1}{2\times\sqrt{2x+4}}\times 2 - 0}\right)$$

$$= \lim_{x\to 0}(\sqrt{2x+4}\ \sec^2 x)$$

$$= \sqrt{2\times 0+4}\times(\sec^2 0)$$

$= 2 \times 1$

$= 2$

23. The negation of the statement "For all real numbers x and y, x + y = y + x" is

(A) for some real numbers x and y, $x + y = y + x$

(B) for some real numbers x and y, $x + y \neq y + x$

(C) for some real numbers x and y, $x - y = y - x$

(D) for all real numbers x and y, $x + y \neq y + x$

Ans (B)

Negation : For some real numbers x and y, $x + y \neq y + x$

24. Let $f : [2, \infty) \rightarrow R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

(A) $[1, \infty)$ (B) $(1, \infty)$ (C) $[5, \infty)$ (D) $(-\infty, \infty)$

Ans (A)

Let $f(x) = y$

$x^2 - 4x + 5 = y$

$x^2 - 2 \cdot x \cdot 2 + 4 + 1 = y$

$x^2 - 2 \cdot x \cdot 2 + 4 = y - 1$

$(x - 2)^2 = y - 1$

$x - 2 = \sqrt{y-1}$

$x = \sqrt{y-1} + 2$

$\therefore\ y - 1 \geq 0$

$y \geq 1$

Range is $[1, \infty)$

25. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that P(A) = 2P(B) = 3P(C), then P(B) is equal to

(A) $\frac{2}{11}$ (B) $\frac{3}{11}$ (C) $\frac{4}{11}$ (D) $\frac{1}{11}$

Ans (B)

P(A) = 2P(B) = 3P(C)

P(A) = 2P(B)

$P(C) = \frac{2}{3}P(B)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$

$-P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$1 = 2\,P(B) + P(B) + \frac{2}{3}P(B) - 0 - 0 - 0 + 0$

$1 = P(B)\left[3 + \frac{2}{3}\right]$

$1 = P(B)\left(\frac{11}{3}\right)$

$P(B) = \frac{3}{11}$

26. If a relation R on the set {1, 2, 3} be defined by R = {(1, 1)}, then R is

(A) Reflexive and transitive (B) Symmetric and transitive

(C) Only symmetric (D) Reflexive and symmetric

Ans (B)

R = {(1, 1)} on a set {1, 2, 3}

R is symmetric and Transitive

27. The value of $\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right)$ is

(A) 1 (B) –1 (C) Does not exist (D) 0

Ans (D)

$\cos\left(\sin^{-1}\frac{\pi}{3} + \cos^{-1}\frac{\pi}{3}\right) = \cos\frac{\pi}{2} = 0$

28. If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then A^4 is equal to

(A) 2A (B) I (C) 4A (D) A

Ans (B)

$$A^2 = AA = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = I$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = I$$

$A^4 = A^3A = I \times A = I$

29. If A = {a, b, c}, then the number of binary operations on A is

(A) 3^6 (B) 3^3 (C) 3^9 (D) 3

Ans (C)

A = {a, b, c}

The number of binary operations are $n^{n^2} = 3^{3^2} = 3^9$

30. The domain of the function defined by $f(x) = \cos^{-1}\sqrt{x-1}$ is

(A) [0, 2] (B) [–1, 1] (C) [0, 1] (D) [1, 2]

Ans (D)

$f(x) = \cos^{-1}\sqrt{x-1}$

$-1 \le \cos^{-1} x \le 1$

and $-1 \le \sqrt{x-1} \le 1$

$0 \le (x - 1) \le 1$

$1 \le x \le 2$

31. If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$ then

(A) f(2) = 0 (B) f(0) = 0 (C) f(–1) = 0 (D) f(1) = 0

Ans (B)

$$f(0) = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

by |skew - symmetric matrix|

f(0) = 0

32. If A and B are square matrices of same order and B is a skew symmetric matrix, then $A'BA$ is

(A) Null matrix (B) Diagonal matrix

(C) Skew symmetric matrix (D) Symmetric matrix

Ans (C)

Given

$B = -B'$

Now $(A'BA)' = (BA)'(A')' \left[\because (AB)' = B'A'\right]$

$= A'B'A$

$= -A'BA$

33. If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the matrix A is

(A) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ (B) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$ (C) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ (D) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

Ans (A)

We know that $A^{-1}A = AA^{-1} = I$ or $BA = AB = I$

Let $B = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \Rightarrow |B| = 4 - 3 = 1$

$\text{adj} B = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

$B^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

$\therefore A = B^{-1} = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$

34. If $f(x) = \begin{cases} \dfrac{1 - \cos Kx}{x \sin x}, & \text{if } x \neq 0 \\ \dfrac{1}{2}, & \text{if } x = 0 \end{cases}$ is continuous at x = 0, then the value of K is

(A) 0 (B) ± 2 (C) ± 1 (D) $\pm\dfrac{1}{2}$

Ans (C)

Given

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \left(\dfrac{1 - \cos Kx}{x \sin x} \right) = \dfrac{1}{2}$

By L'H rule

$\lim_{x \to 0} \left(\dfrac{\sin Kx \cdot K}{x \cos x + \sin x} \right) = \dfrac{1}{2}$

By L′H rule

$\lim_{x \to 0} \left(\dfrac{\cos Kx \cdot K^2}{-x \sin x + \cos x + \cos x} \right) = \dfrac{1}{2}$

$\dfrac{1 \cdot K^2}{0 + 1 + 1} = \dfrac{1}{2} \Rightarrow K^2 = 1$

K = ± 1

35. If $a_1\ a_2\ a_3 \ldots\ldots a_9$ are in A.P. then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(A) $a_1 + a_9$ (B) $\log_e(\log_e e)$ (C) 1 (D) $\dfrac{9}{2}(a_1 + a_9)$

Ans (B)

Let a be first term and d be common difference

$$GE = \begin{vmatrix} a & a+d & a+2d \\ a+3d & a+4d & a+5d \\ a+6d & a+7d & a+8d \end{vmatrix}$$

by applying $C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_2$

$$= \begin{vmatrix} a & d & d \\ a+3d & d & d \\ a+6d & d & d \end{vmatrix}$$

$= 0$

$= \log(\log_e e)$ [$\because$ log 1 = 0]

36. If A is a square matrix of order 3 and |A| = 5, then |A adj A| is

(A) 125 (B) 25 (C) 625 (D) 5

Ans (A)

|A adj A| = |A| |adj A|

$= |A||A|^{3-1}$

$= 5 \cdot 5^2$

$= 125$

37. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'\left(\sqrt{3}\right)$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{3}}$ (C) $-\frac{1}{\sqrt{3}}$ (D) $-\frac{1}{2}$

Ans (A)

Put x = tan θ ⇒ θ = $\tan^{-1}x$

$$f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$f(x) = 2\tan^{-1} x$

$$f'(x) = \frac{2}{1+x^2}$$

$$f'\left(\sqrt{3}\right) = \frac{2}{1+3} = \frac{1}{2}$$

38. The right hand and left hand limit of the function $f(x) = \begin{cases} \frac{e^{1/x}-1}{e^{1/x}+1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$ are respectively

(A) 1 and –1 (B) –1 and –1 (C) –1 and 1 (D) 1 and 1

Ans (D)

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}\right)$$

By dividing both numerator and denominator by $e^{\frac{1}{x}}$

$$= \lim_{x\to 0}\left(\frac{1-e^{-\frac{1}{x}}}{1+e^{-\frac{1}{x}}}\right)$$

$$= \lim_{x\to 0}\left(\frac{1-0}{1+0}\right)\left[\begin{array}{c} \because e^{\infty} = \infty \\ e^{-\infty} = \frac{1}{\infty} = 0 \end{array}\right]$$

$= 1$

$\therefore$ LHL = RHL = 1

39. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is

(A) -2^{y-x} (B) 2^{x-y} (C) $\frac{2^y-1}{2^x-1}$ (D) 2^{y-x}

Ans (A)

We know that,

If $a^x + a^y = a^{x+y}$ then $\frac{dy}{dx} = -a^{y-x}$

$\therefore \frac{dy}{dx} = -2^{y-x}$

40. If the curves $2x = y^2$ and $2xy = k$ intersect perpendicularly, then the value of K^2 is

(A) $2\sqrt{2}$ (B) 2 (C) 8 (D) 4

Ans (C)

$2x = y^2$ …(1)

$2xy = k$ …(2)

Solve (1) and (2)

(2) $\Rightarrow y^3 = k$

$y = k^{\frac{1}{3}}$

(1) $\Rightarrow x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$

$(x, y) = \left(\frac{k^{\frac{2}{3}}}{2}, k^{\frac{1}{3}}\right)$

Differentiate (1) w.r.t x

$y' = \frac{1}{y}$

$m_1 = \frac{1}{k^{\frac{1}{3}}}$

Differentiate (2) w.r.t x

$y' = -\frac{y}{x}$

$m_2 = \frac{2k^{\frac{1}{3}}}{k^{\frac{2}{3}}} = \frac{-2}{k^{\frac{1}{3}}}$

Given

$m_1 m_2 = -1$

$\frac{1}{k^{\frac{1}{3}}} \times \frac{-2}{k^{\frac{1}{3}}} = -1$

$k^{\frac{2}{3}} = 2$

$k^2 = 8$

41. If $(xe)^y = e^x$, then $\frac{dy}{dx}$ is

(A) $\frac{1}{(1+\log x)^2}$ (B) $\frac{\log x}{(1+\log x)}$ (C) $\frac{e^x}{x(y-1)}$ (D) $\frac{\log x}{(1+\log x)^2}$

Ans (D)

$(xe)^y = e^x$

$\Rightarrow y(\log x + 1) = x$

$\Rightarrow y = \frac{x}{\log x + 1}$

$\therefore \frac{dy}{dx} = \frac{\log x}{(\log x + 1)^2}$

42. If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2 \frac{d^2y}{dx^2}$ is

(A) n(n + 1)y (B) $x\frac{dy}{dx} + y$ (C) y (D) 6n(n + 1)y

Ans (A)

$y = 2x^{n+1} + 3x^{-n}$

$\Rightarrow \frac{dy}{dx} = 2(n+1)x^n - 3nx^{-n-1}$

$\Rightarrow \frac{d^2y}{dx^2} = 2n(n+1)x^{n-1} + 3n(n+1)x^{-n-2}$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)\left[2x^{n+1} + \frac{3}{x^n}\right]$

$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)y$

43. The value of $\int \frac{1+x^4}{1+x^6} dx$ is

(A) $\tan^{-1} x + \frac{1}{3}\tan^{-1} x^3 + C$ (B) $\tan^{-1} x - \frac{1}{3}\tan^{-1} x^3 + C$

(C) $\tan^{-1} x + \frac{1}{3}\tan^{-1} x^2 + C$ (D) $\tan^{-1} x + \tan^{-1} x^3 + C$

Ans (A)

$\int \frac{1+x^4}{1+x^6} = \int \frac{1+x^4-x^2+x^2}{(1+x^2)(1-x^2+x^4)} dx$

$= \int \frac{(1-x^2+x^4)}{(1+x^2)(1-x^2+x^4)} + \frac{x^2}{(1+x^2)(1-x^2+x^2)} dx$

$$= \int \left(\frac{1}{1+x^2} + \frac{x^2}{1+x^6} \right) dx$$

$$= \int \left(\frac{1}{1+x^2} + \frac{1}{3} \frac{3x^2}{1+(x^3)^2} \right) dx$$

$$= \tan^{-1} x + \frac{1}{3} \tan^{-1}(x^3) + C$$

44. The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

(A) 1 (B) $\frac{1}{e}$ (C) $-\frac{1}{e}$ (D) e

Ans (B)

$y = \frac{\log x}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1-\log x}{x^2}$

$\frac{dy}{dx} = 0$

$\Rightarrow 1 - \log x = 0$

$\Rightarrow x = e$

$\therefore\ y_{max} = \frac{1}{e}$

45. If the side of a cube is increased by 5%, then the surface area of a cube is increased by

(A) 60% (B) 6% (C) 20% (D) 10%

Ans (D)

$A = 6x^2$ $\qquad \frac{dx}{dt} = \frac{5x}{100}$

$\frac{dA}{dt} = 12x \frac{dx}{dt} = 12x \times \frac{5x}{100} = \frac{60\,x^2}{100}$

$\Rightarrow\ = \frac{10}{100} \times 6x^2 = \frac{10}{100} A$

$\therefore$ 10%

46. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1} x\, dx$ is

(A) $\frac{\pi}{2}$ (B) 1 (C) $\frac{\pi^2}{2}$ (D) π

Ans (A)

$$\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1} x\, dx = x\cos^{-1} x \Big|_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x}{\sqrt{1-x^2}} dx$$

$$= \frac{1}{2}\cos^{--1}\left(\frac{1}{2}\right) + \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right) - \frac{1}{2} \cdot 2\sqrt{1-x^2}\Big]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$=\frac{1}{2}\cdot\frac{\pi}{3}+\frac{1}{2}\cdot\frac{2\pi}{3}-0$$

$$=\frac{\pi}{6}+\frac{\pi}{3}=\frac{\pi}{2}$$

47. If $\int\frac{3x+1}{(x-1)(x-2)(x-3)}dx$ = A log |x – 1| + B log |x – 2| + C log |x – 3| + C, then the values of A, B and C are respectively,

(A) 2, –7, –5 (B) 5, –7, 5 (C) 2, –7, 5 (D) 5, –7, –5

Ans (C)

$$\frac{3x+1}{(x-1)(x-2)(x-3)}=\frac{A}{x-1}+\frac{B}{x-2}+\frac{C}{x-3}$$

$$\Rightarrow 3x+1=A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$$

$x = 1 \Rightarrow A = 2$, $x = 2 \Rightarrow B = -7$, $x = 3 \Rightarrow C = 5$

48. The value of $\int e^{\sin x}\sin 2x\,dx$ is

(A) $2\,e^{\sin x}(\sin x + 1) + C$ (B) $2\,e^{\sin x}(\cos x + 1) + C$

(C) $2\,e^{\sin x}(\cos x - 1) + C$ (D) $2\,e^{\sin x}(\sin x - 1) + C$

Ans (D)

$$\int e^{\sin x}\sin 2x\,dx=2\int e^{\sin x}\sin x\cos x\,dx = =2\int t\,e^t dt$$

$$=2\left[te^t-e^t\right]+c \qquad \sin x = t$$

$$=2[\sin x-1]e^{\sin x}+c \qquad \Rightarrow \cos x\,dx=dt$$

49. The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

(A) $\frac{4}{3}$ sq. units (B) $\frac{3}{4}$ sq. units (C) $\frac{8}{3}$ sq. units (D) $\frac{16}{3}$ sq. units

Ans (A)

$y^2 = 8x$ and $y = 2x$

$\Rightarrow 4x^2 = 8x$

$\Rightarrow x^2 - 2x = 0$

$\Rightarrow x = 0, 2$

$$RA = \int_0^2\left(\sqrt{8}\cdot x^{\frac{1}{2}}-2x\right)dx$$

$$=\left.2\sqrt{2}\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}}-x^2\right]_0^2=\frac{4\sqrt{2}}{3}2^{\frac{3}{2}}-2^2-0$$

$$=\frac{4\sqrt{2}}{3}2\sqrt{2}-4=\frac{16}{3}-4$$

$$=\frac{4}{3}\text{ sq. units}$$

50. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx$ is

(A) 0 (B) 1 (C) –2 (D) 2

Ans (B)

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x} dx \qquad ...(1)$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-\frac{\pi}{2}-x\right)}{1+e^{\frac{\pi}{2}-\frac{\pi}{2}-x}} dx$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^{-x}} dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{e^x \cos x}{1+e^x} dx \qquad ...(2)$$

$$(1) + (2) \Rightarrow 2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{(e^x+1)\cos x}{e^x+1} dx$$

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x dx = \sin x \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$2I = \sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)$$

$2I = 1 - (-1) = 2$

$\therefore I = 1$

51. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$ is

(A) $\frac{\pi}{4}\log 2$ (B) $\frac{1}{2}$ (C) $\frac{\pi}{8}\log 2$ (D) $\frac{\pi}{2}\log 2$

Ans (C)

$$\int_0^1 \frac{\log(1+x)}{1+x^2} dx \quad \left| \begin{array}{l} x = \tan\theta \\ dx = \sec^2\theta \, d\theta \end{array} \right.$$

$$= \int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta$$

$$= \frac{\pi}{8}\log 2$$

52. The general solution of the differential equation $x^2 dy - 2xy dx = x^4 \cos x \, dx$ is

(A) $y = x^2 \sin x + c$ (B) $y = \sin x + cx^2$

(C) $y = \cos x + cx^2$ (D) $y = x^2 \sin x + cx^2$

Ans (D)

$x^2 \, dy - 2xy \, dx = x^4 \cos x \, dx$

$$\frac{dy}{dx} = \frac{x^4 \cos x + 2xy}{x^2}$$

$$\frac{dy}{dx} - \frac{2}{x} y = x^2 \cos x$$

$$IF = e^{\int -\frac{2}{x} dx} = e^{-2\log x} = \frac{1}{x^2}$$

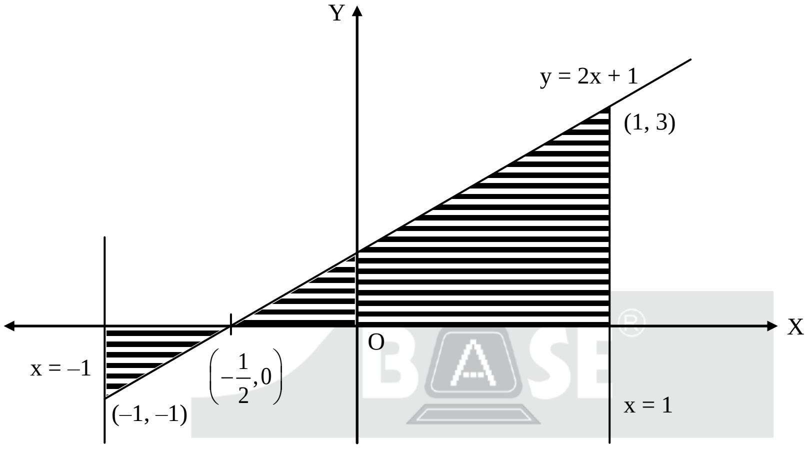
General solution is $y\left(\frac{1}{x^2}\right) = \int \frac{1}{x^2}\left(x^2 \cos x\right) dx + c$

$$\frac{y}{x^2} = \sin x + c$$

53. The area of the region bounded by the line y = 2x + 1, x-axis and the ordinates x = –1 and x = 1 is

(A) 2 (B) $\frac{5}{2}$ (C) 5 (D) $\frac{9}{4}$

Ans (B)



Area bounded by y = 2x + 1 with x-axis $= \frac{1}{2}\left(\frac{1}{2}\right)(1) + \frac{1}{2}\left(\frac{3}{2}\right)(3)$

$$= \frac{1}{4} + \frac{9}{4}$$

$$= \frac{10}{4}$$

$$= \frac{5}{2} \text{ sq. units}$$

54. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1 y = (c_2 + c_3)e^{x+c_4}$ is

(A) 2 (B) 3 (C) 4 (D) 1

Ans (D)

$$c_1 y = \left(c_2 + c_3\right)e^{x+c_4}$$

$$y = \left(\frac{c_2 + c_3}{c_1} e^{c_4}\right)e^x$$

$y = Ae^x$

Order = number of arbitrary constants

= 1

55. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\frac{\theta}{2}$ is

(A) $\frac{|\vec{a}+\vec{b}|}{2}$ (B) $\frac{|\vec{a}-\vec{b}|}{2}$ (C) $|\vec{a}-\vec{b}|$ (D) $|\vec{a}+\vec{b}|$

Ans (B)

$|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

$= 2(1-\cos\theta)$ $(\because |\vec{a}| = |\vec{b}| = 1)$

$|\vec{a}-\vec{b}|^2 = 2\left(2\sin^2\frac{\theta}{2}\right)$

$\sin\frac{\theta}{2} = \frac{|\vec{a}-\vec{b}|}{2}$

56. The curve passing through the point (1, 2) given that the slope of the tangent at any point (x, y) is $\frac{2x}{y}$ represents

(A) Parabola (B) Ellipse (C) Hyperbola (D) Circle

Ans (C)

Slope $= \frac{dy}{dx} = \frac{2x}{y}$

$\Rightarrow$ y dy = 2x dx

$\Rightarrow \int y\,dy = \int 2x\,dx + A$

$\Rightarrow \frac{y^2}{2} = x^2 + A \Rightarrow \frac{y^2}{2} - x^2 = A$

$\Rightarrow$ curve is hyperbola

57. The two vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}+5\hat{k}$ represent the two sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of a ΔABC. The length of the median through A is

(A) 14 (B) 7 (C) $\sqrt{14}$ (D) $\frac{\sqrt{14}}{2}$

Ans (C)

$\overrightarrow{AB} = (1, 1, 1)$ $\overrightarrow{AC} = (1, 3, 5)$

$\overrightarrow{BC} = (0, 2, 4)$

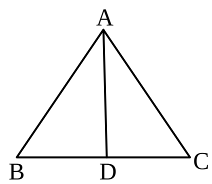
$\overrightarrow{BD} = (0, 1, 2)$

$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= (1, 2, 3)$

$|\overrightarrow{AD}| = \sqrt{1+4+9}$

$= \sqrt{14}$



58. The point (1, –3, 4) lies in the octant

(A) Third (B) Fourth (C) Eighth (D) Second

Ans (B)

(1, –3, 4)

Fourth octant

59. If the vectors $2\hat{i}-3\hat{j}+4\hat{k}$, $2\hat{i}+j-\hat{k}$ and $\lambda\hat{i}-\hat{j}+2\hat{k}$ are coplanar, then the value of λ is

(A) –5 (B) –6 (C) 5 (D) 6

Ans (D)

Vectors are coplanar

$$\Rightarrow \begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$$

$\Rightarrow 2(1) + 3(4 + \lambda) + 4(-2 - \lambda) = 0$

$\Rightarrow 2 + 12 + 3\lambda - 8 - 4\lambda = 0$

$\Rightarrow \lambda = 6$

60. If $\left|\vec{a}\times\vec{b}\right|^2+\left|\vec{a}\cdot\vec{b}\right|^2=144$ and $|\vec{a}|=6$, then $|\vec{b}|$ is equal to

(A) 3 (B) 2 (C) 4 (D) 6

Ans (B)

$\left|\vec{a}\times\vec{b}\right|^2+\left(\vec{a}.\vec{b}\right)^2=144$

$\Rightarrow |\vec{a}|^2\,|\vec{b}|^2=144$

$\Rightarrow |\vec{b}|^2=\dfrac{144}{36}=4$

$\Rightarrow |\vec{b}|=2$

* * *